# Tetrascrits Cannot Fill a Rectangle

Version 6, April 2026
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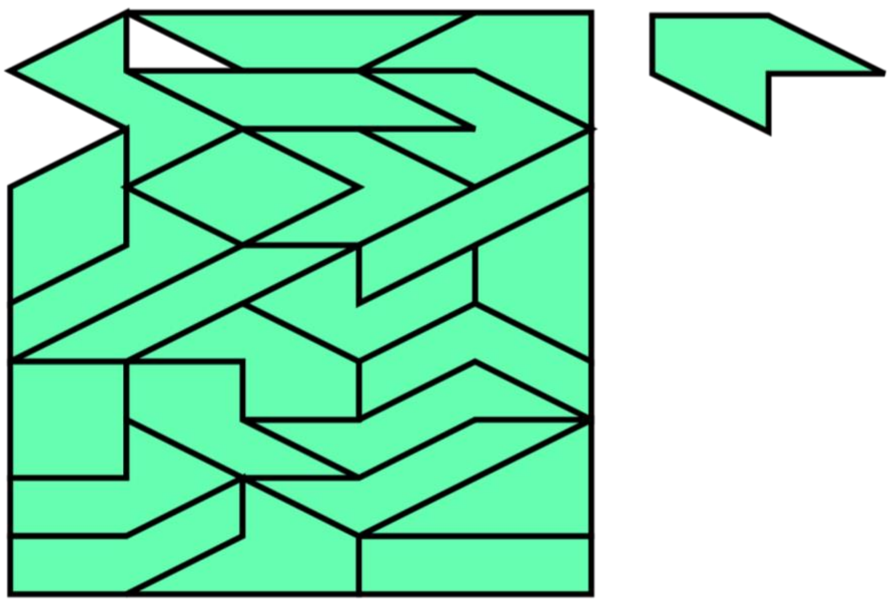
## 1. The Challenge

A question was posted on Facebook Puzzle-Fun, asking for a proof that a certain puzzle is impossible. It was posted on 26 December 2025 by Brendan Owen, who had already made an exhaustive search for solutions and found none. By the nature of the puzzle, it seemed likely there would be solutions, so it was surprising there are none. He asked for some way, such as a parity argument, to show there can be no solutions. The posting, reformatted, is below.



"I am trying to fit these pieces into a square. I did a complete search and the most I could tile was 24 pieces. Can anyone see a parity reason why the last piece cannot be placed?" — Brendan Owen

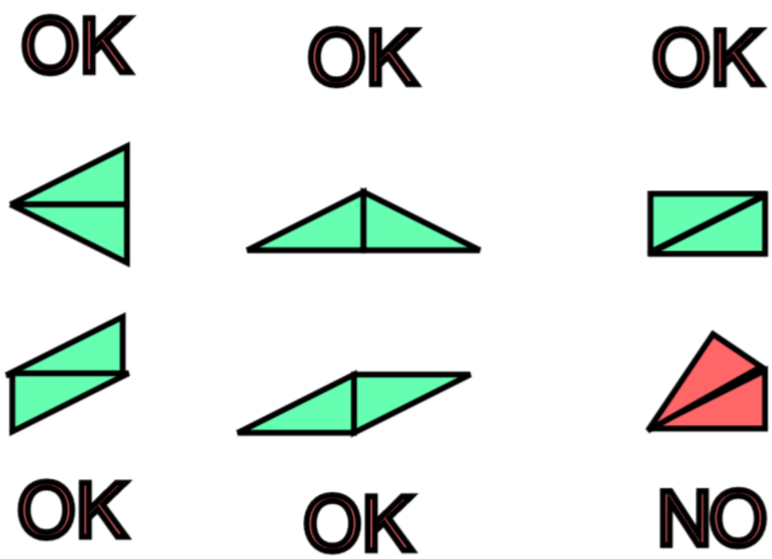
## 2. An Answer Is Found

I found a proof that the rectangle in the Facebook post cannot be filled with the given pieces. It is not especially simple, and involves coloring, but other considerations also. Although it is a bit involved, it does not use extensive case-by-case analysis, and does not rely on computer searches.

## 3. Definitions and Descriptions

In two-dimensional geometric puzzles, a *polyform* is a puzzle piece made by attaching copies of a basic, polygonal shape to each other. The most common is probably polyominoes. The pieces discussed here are made by attaching scalene right triangles together. A right triangle has a 90-degree interior angle; the side opposite that angle is the hypotenuse, and the other two other sides are legs. A scalene triangle has all three sides of different length.

In this puzzle, all of the scalene right triangles are *isomorphic*; they are the same shape and size. They are joined along edges of equal length, and the areas do not overlap. This allows for six types of seam line, but one of those six is disallowed: joining hypotenuse to hypotenuse must create only a rectangle, not a kite shape.




The pieces are "two-sided"; they can be picked up, flipped over, and are still considered the same piece. That is, the mirror image of a piece is considered to be the same piece.

I have not found a standard name for these pieces in the literature, so I named them ***polyscrits***, for POLY SCalene RIght Triangles.

The *order* of a polyform is the number of basic shapes glued together. For polyscrits, we have:

| Order | Name | Number of distinct pieces |
|---|---|---|
| 1 | monoscrit | 1 |
| 2 | discrit or biscrit | 5 |
| 3 | triscrit | 6 |
| 4 | tetrascrit | 25 |
| 5 | pentascrit | 57 ? |

The puzzle in question here uses tetrascrit pieces, of which there are 25 distinct shapes. As the table above implies, I am unsure how many polyscrits there are beyond order 4 (but see the section "Polyscrit Counts" near the end).

In most polyform puzzles, a given set of pieces is to be placed without overlap into an enclosure, or onto an area, called the *board*. For the puzzle at hand, the set of 25 tetrascrit pieces is to be placed on a rectangular board.

All the component triangles in all the pieces are aligned, so that the pieces follow the same joining rule as in making pieces. We follow the example in the challenge: the long leg of each triangle is oriented horizontally, and the short leg is oriented vertically. One can imagine a different puzzle, in which pieces can be rotated a quarter turn, but that is disallowed here. Note that this means vertical and horizontal are especially distinct; pieces may be translated, reflected, or rotated 180 degrees, but not rotated 90 degrees!

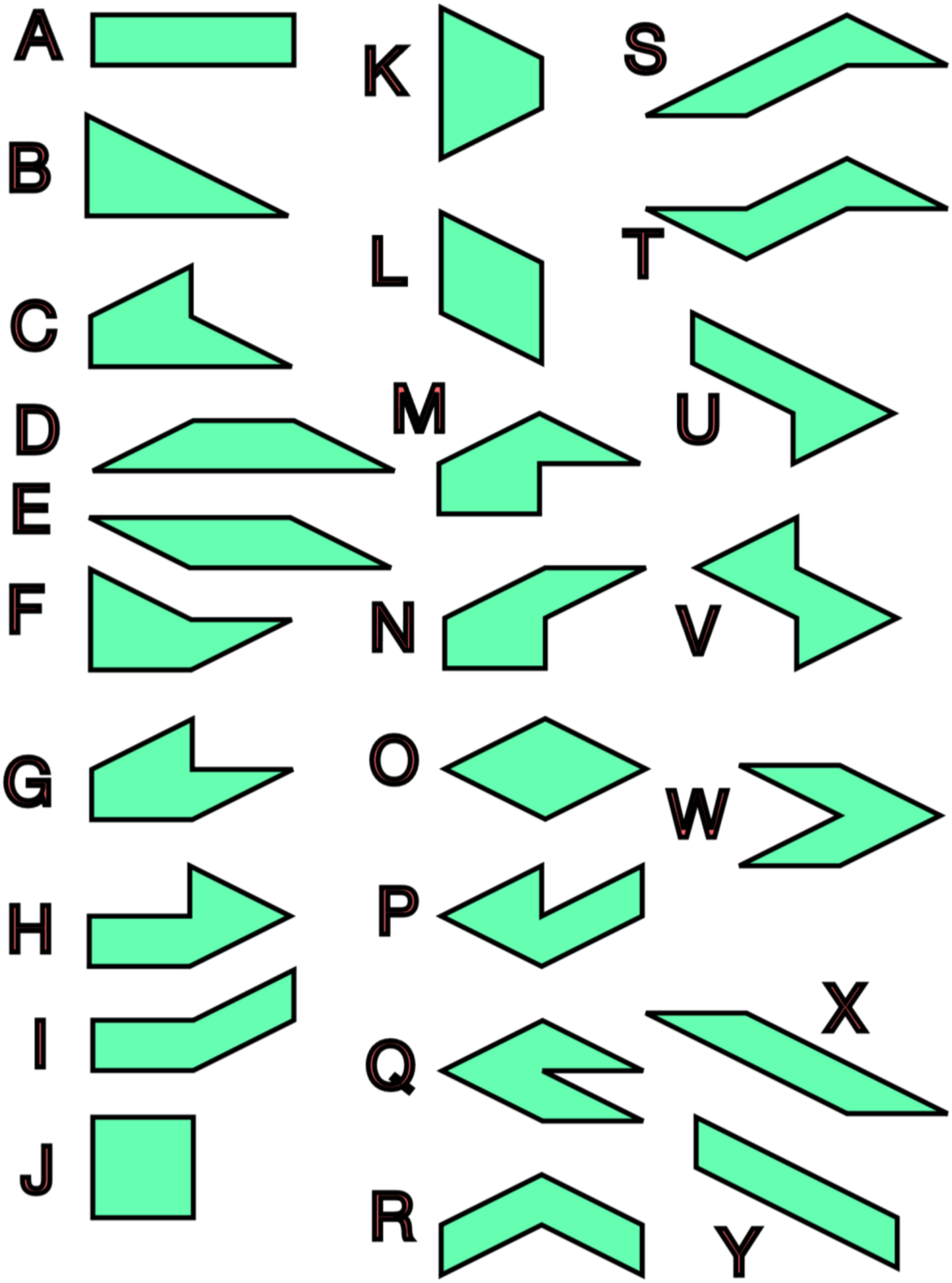
A scalene right triangle can have any ratio of its legs, except of course 1:1, when it is isosceles, not scalene. The entire analysis in this paper can apply to tetrascrit pieces made with any* given leg ratio. Yet, the original challenge uses triangles with a ratio of 2:1, the simplest whole number ratio other than 1:1. Using 2:1 has a particular appeal as a puzzle: it allows for a square board. For simplicity, we use triangles with ratio 2:1.

* However, the ratio √3:1 is special. As Brendan Owen noted in a comment to his Facebook post, this makes the triangles 30-60-90 degrees. Then, tetrascrits L and O have the same size and angles. And the pieces can be arranged on an equilateral triangular grid. If we discard either L or O, we have a set of 24 distinct pieces that can make various shapes, such as a large hexagon as Brendan Owen shows. Roel Huisman posted other shapes that this set makes. 30-60-90 degree triangles are also the basic unit of *polydrafter* pieces (see Wikipedia).

Also of note, the ratio 2:1 basic shape is called a *dom* in the puzzle and tiling community, for half a domino cut along a diagonal. However, *polydoms* are allowed to join forming kites, and to join halfway along the longer leg. This creates many more pieces, and allows the grid on which pieces are aligned to change. Polyscrits are a very restricted subset of polydoms.

In this analysis, we use the short leg of component triangles as the basic unit of *length*. Thus, the component triangles are 2 units wide and 1 unit high. The board is 10 units by 10 units. The board can be thought of as a rectangular grid; each cell is 2 units wide and 1 unit high. So, the board is 5 cells wide and 10 cells high. We use the component triangle as the basic unit of *area*, and call it a *half-cell*. Each puzzle piece has an area of 4 half-cells, and the board has an area of 100 half-cells.

I did not find names for the tetrascrits in the literature, so I gave them names, A through Y.


A
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Y

## 4. Vertical Boundaries

The proof begins here. It is a proof by contradiction; we assume that a solution exists that places all 25 tetrascrit pieces on the 10-by-10 board, examine what conditions such a solution must satisfy, and find that it is impossible to satisfy those conditions.

Consider the left and right sides of the board. Each is 10 units long, for a total of 20 units. The area of the board exactly matches the total area of the pieces, so there are no holes or gaps, and the 20 units of vertical border must be made of vertical sides of pieces. 16 pieces have a section of their perimeter that can contribute toward the needed 20 units. Some pieces, such as V, have a vertical section that cannot be on the border because part of the piece projects beyond that section.

It is important to note that no pieces are wide enough to span the board. Such pieces, if they had vertical sections on both sides, could contribute twice. Also note that a piece may contact the border at a point, contributing zero vertical length; we are not concerned with such pieces now.

The following table gives parameters of the 16 pieces that can contribute to the vertical borders.

| **Piece** | **V units on longer side** | **V units on other side** | **Area in border column** |
|---|---|---|---|
| A | 1 | 1 | 2 |
| B | 2 | 0 | 3 |
| C | 1 | 0 | 3 |
| F | 2 | 0 | 3 |
| G | 1 | 0 | 3 |
| H | 1 | 0 | 2 |
| I | 1 | 1 | 2 |
| J | 2 | 2 | 4 |
| K | 3 | 1 | 4 |
| L | 2 | 2 | 4 |
| M | 1 | 0 | 3 |
| N | 1 | 0 | 3 |
| P | 1 | 0 | 2 |
| R | 1 | 1 | 2 |
| U | 1 | 0 | 2 |
| Y | 1 | 1 | 2 |
| total | 22 | 9 | 44 |

We are checking to see whether we have enough vertical contributions to make the 20 units of the board, so if a piece has vertical units on both sides, we show the larger side in the column "V units on longer side". The other columns are needed for analysis a bit later.

The total available is 22 units, 2 more than needed. This allows the possibility of a solution, and in fact means we cannot use all of these contributions in a solution. We must find a way to reduce the total contribution to 20.

## 5. Options to Adjust the Vertical Total

There are two ways to reduce the total vertical border contributions to 20. One is to use the smaller side of pieces that have a vertical section on both sides. The other way is to omit pieces from being on the border.

First consider using the smaller side. The vertical length of that smaller side is shown in the column "V units on other side" in the table above. For 9 pieces, the other side is 0; that is, there is no vertical length available, so flipping these pieces over is equivalent to omitting them from the border; such interior pieces are considered later. For 6 pieces (A I J L R Y), the other side has the same vertical length, so using the other side does not help.

For the one other piece (K), one side is 3 units and the other side is 1 unit. Flipping piece K to use its 1-unit side along the border reduces the total by 2, achieving our goal, a total of 20. Note that K is still one of the border pieces, so in any solution K must be on the border.

Now consider omitting pieces. We can omit a piece with 2 vertical units, such as any of (B F J L). But, below, we will find we cannot omit B or F. We are left with omitting J or L.

Or, we can omit any two pieces with 1 vertical unit, namely the 11 pieces (A C G H I M N P R U Y).

There is a further limitation on what pieces can be placed along the vertical border. Consider the areas in the left and right columns of the board. Each column is 10 units high, so it contains 10 2-by-1 cells, thus 20 half-cells, for a total area of 40 half-cells. Some of these half-cells might be filled by pieces that do not touch the border, but 40 is an upper limit on the area that vertical border pieces can occupy in those two columns.

The column "Area in border column" in the table above is the area that each potential border piece occupies in the columns at the edges of the board. The total is 44, 4 more than there is space on the board. For any solution, we must somehow reduce this area total. The only chance we have is the options above. Do any of those achieve this reduction?

Flipping K does not help; it still occupies 4 half-cells. Further, if K is flipped, the total of vertical units is now 20, but we must still remove 4 half-cells of area from the border columns, and doing so would reduce the vertical units below the required 20. Therefore, K must contribute its length 3 side.

Omitting B or F does not help enough; each of them occupies only 3 half-cells next to the border. However, omitting J or L, each of which occupies 4 half-cells, does work. Thus, B and F must be on the border (like K), but we can omit J or L.

The 11 pieces (A C G H I M N P R U Y) each occupy 2 or 3 half-cells, so omitting any two of them works. If two are omitted that reduce the total to less than 40, that is OK, because the 1 or 2 half-cells less than 40 can be filled by pieces that touch the border only at a point.

A final consideration is flipping any of the pieces that have a vertical section on both sides. Would that reduce the border column total area? Those pieces are (A I J L R Y). Examining their shapes, we find flipping makes no difference in this regard.

To summarize, the available options are:

- Omit J or L.
- Omit any 2 of (A C G H I M N P R U Y).
- B, F and K must be on a vertical border. K must contribute 3 units.

## 6. Columns of Alternating Color

Color the board in vertical bands of alternating color. The outermost columns and the middle column are black, while the other two columns are white. The black area is 60 half-cells, and the white area is 40 half-cells.

When a piece is placed on the board, imagine that its four component triangles take on the color of the board's band under them. If a piece is moved one column left or right, its triangles all change color. If a piece is moved up or down, the color of its triangles is unchanged. If a piece is flipped over, the color of its triangles changes accordingly.

It turns out that, for each of the 25 tetrascrits, the split between the number of black triangles and white triangles remains constant as the piece is translated or reflected. This classifies the pieces as shown in the table below.

| Split | Absolute difference | Piece count | Piece names |
|---|---|---|---|
| 4 – 0 | 4 | 3 | J K L |
| 3 – 1 | 2 | 6 | B C F G M N |
| 2 – 2 | 0 | 16 | A D E H I O P Q R S T U V W X Y |

Let us consider what pieces can make up the 40 white half-cells in a solution.

The 16 pieces that split 2-2 contribute a total of 32 white half-cells, leaving 8 to go.

The 6 pieces that split 3-1 contribute *at least* a total of 6, leaving 2 to go.

The 2 white half-cells remaining cannot be occupied by any of the pieces that split 4-0, because 4 exceeds the remaining area (2). Thus, exactly one of (B C F G M N) must be 3 white and 1 black.

## 7. Options to Adjust the White Area

Let us consider together the options to build the 20 units of vertical border, and the options to fill the 40 half-cells of white area. The table below summarizes the status.

| | Split 2-2 | Split 3-1 | Split 4-0 |
|---|---|---|---|
| Must be on border | — | B F | K |
| Can omit 1 from border | — | — | J or L |
| Can omit 2 from border | A H I P R U Y | C G M N | — |
| Not on border | D E O Q S T V W X | — | — |

What piece(s) can we move off the border, and what color splits can we assign, to get a solution? The 16 pieces split 2-2 are already accounted for. The pieces that must be on the border cannot be moved. And pieces that split 3-1 or 4-0 are majority black if they are on the border. We need to find a way to increase the total white area from 38 half-cells to 40. Can we omit J or L from the border, moving one of these to a white band? No, that would add 4 half-cells to white, too much.

The only possibility left is to omit two of (C G M N) from the border. When on the border, each of these pieces has 3 black half-cells and 1 white. One of these pieces is moved so it has 3 half-cells on white and 1 on black, increasing the white area by 2 half-cells, as needed. The other piece is moved so its 1 white half-cell stays on white, and its 3 black half-cells now fall on the middle band, so they remain black.

In summary, we now know:

- 12 pieces (A B F H I J K L P R U Y) must be on the border
- Piece K must have its side of 3 vertical units on the border
- 2 of the 4 pieces (C G M N) must be on the border
- 2 of the 4 pieces (C G M N) must be in the interior (not on the border)
- 9 pieces (D E O Q S T V W X) must be in the interior
- "The border" 20 vertical units is the sum of the left and right sides of the board. Pieces on the border must be partitioned so they contribute 10 units to each side. However, that partitioning is not considered here, as it is not needed for the proof.

## 8. Pairing interior vertical units

Consider the vertical units on the pieces, now including those behind a projection such as on V. Each unit must either be on the (left or right) vertical boundary, or else it must abut another piece at one of its vertical units. Most of the pieces that are on the vertical boundary have vertical sides that fall in the interior of the board. The table below summarizes these.

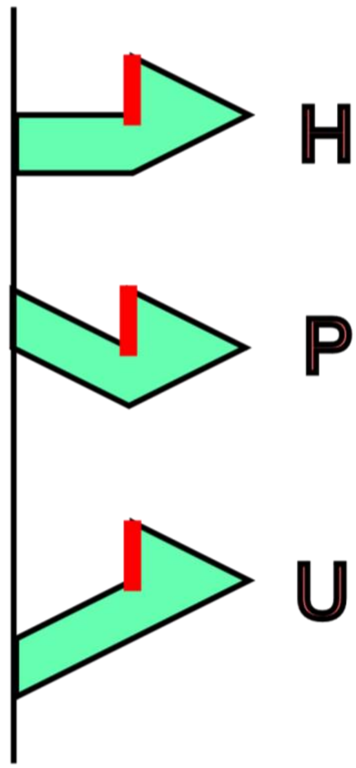
| **Border piece** | **Interior vertical units** |
|---|---|
| A | 1 @ 2 |
| B | 0 |
| F | 0 |
| H | 1 out-facing @ 1 |
| I | 1 @ 2 |
| J | 2 @ 1 |
| K | 1 @ 1 |
| L | 2 @ 1 |
| P | 1 out-facing @ 1 |
| R | 1 @ 2 |
| U | 1 out-facing @ 1 |
| Y | 1 @ 2 |
| (2 of these 4) C | 1 @ 1 |
| (2 of these 4) G | 1 @ 1 |
| (2 of these 4) M | 1 @ 1 |
| (2 of these 4) N | 1 @ 1 |

Summarizing the number of pieces with each kind of interior vertical units:

| **Interior vertical units** | **Number of pieces** | **Net vertical units** |
|---|---|---|
| 0 | 2 | 0 |
| 1 @ 1 | 3 | 4 @ 1 |
| 1 out-facing @ 1 | 3 | |
| 2 @ 1 | 2 | |
| 1 @ 2 | 4 | 4 @ 2 |

"1 @ 1" means the piece presents 1 interior vertical unit at 1 column away from the edge of the board. "2 @ 1" means 2 interior vertical units, again at 1 column away from the edge. "1 @ 2" means 1 vertical unit, but 2 columns away from the board edge, thus abutting the middle column of the board.

"1 out-facing @ 1" means 1 interior vertical unit, but not facing the interior of the board. Rather, the vertical unit is on the outward facing side of a hook-shape. The diagram below shows where the out-facing unit is on pieces H, P and U.




The three out-facing units cannot* pair with interior pieces. They must pair with vertical units on border pieces, specifically those at one column from the edge of the board ("1 @ 1" or "2 @ 1"). This reduces the number of vertical units presented to the interior of the board. Without this effect, we would expect 3 units from the three "1 @ 1" pieces, and 4 units from the two "2 @ 1" pieces, for a total of 7, that we could call "7 @ 1". However, the out-facing units pair with 3 of these, leaving a net of only 4 ("4 @ 1").

* This assertion deserves further consideration. Perhaps an interior piece can hook around the border piece to gain access to the out-facing unit and pair with it. This seems possible, because we account for only 38 of the 40 half-cells of area in the border columns. Two half-cells will be filled by one or two interior pieces. Suppose those two half-cells are adjacent, and next to a piece with an out-facing unit. A hook-shaped interior piece might occupy those two half-cells, thus pairing with the out-facing unit. But wait, what pieces could be such a hook-shaped piece? A little consideration finds it could be only H, P or U. But H, P and U are already assigned to be border pieces; they cannot be an inside piece.

The total is "4 @ 1" plus "4 @ 2", for a total of 8 vertical units presented to the interior of the board.

None of these 8 units can pair with each other. They are 1 or 2 columns away from the edge of the board, so they cannot reach those on the other side.

The following table shows what vertical units the interior (non-border) pieces have, that might pair with the units presented by the border pieces.

| Interior piece | Vertical units |
|---|---|
| (2 of these 4) C | 2 (1 clear, 1 recessed) |
| (2 of these 4) G | 2 (1 clear, 1 recessed) |
| (2 of these 4) M | 2 (1 clear, 1 recessed) |
| (2 of these 4) N | 2 (1 clear, 1 recessed) |
| D | 0 |
| E | 0 |
| O | 0 |
| Q | 0 |
| S | 0 |
| T | 0 |
| V | 2 (0 clear, 2 recessed) |
| W | 0 |
| X | 0 |

For clarity, we note that some of these vertical units are accessible to a flat surface, namely those on pieces that could be border pieces. Other units are in positions that are "recessed", further back than the outermost point of the piece. For a recessed unit to pair with another unit, there must be a vacancy, a gap, next to the paired unit for the "bump" to fit into. For example, we accounted for 38 half-cells in the border, so there is a gap of 2 half-cells (or two gaps of 1 half-cell each) that such a piece might fill. But in fact, this distinction is not necessary for the proof.

The table below summarizes the vertical units available from interior pieces.

| Available vertical units | Number of pieces | Net vertical units |
|---|---|---|
| 0 | 8 | 0 |
| 2 (1 clear, 1 recessed) | 2 | 4 |
| 2 (0 clear, 2 recessed) | 1 | 2 |

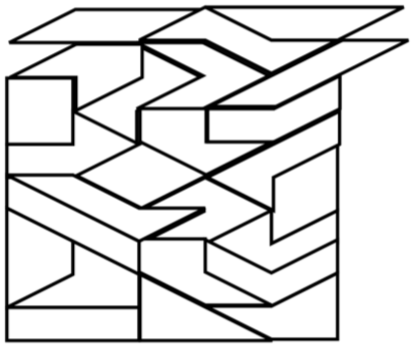
The interior pieces have a total of 6 vertical units available.

## 9. Conclusion

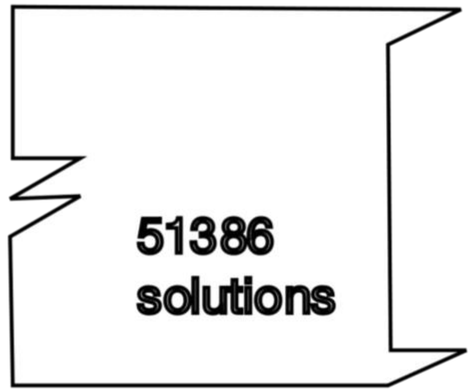
The interior pieces provide 6 vertical units, not enough to pair with the 8 vertical units presented by the border pieces. Therefore, there can be no solution. Q.E.D.

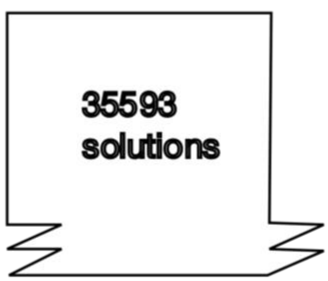
## 10. Inspiration: Perturbed Boards with Solutions

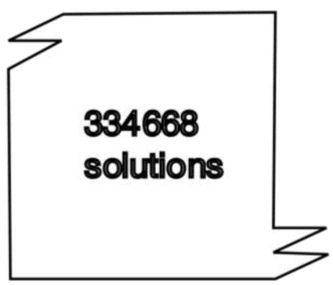
I got inspiration to persist in looking for a proof, and for various approaches I tried, from a test I ran with boards of perturbed shape. I found that many shapes that have two half-cells (but not a full cell) moved from one vertical side to the opposite side, have solutions. In fact, they generally have thousands of solutions. This suggests there is some subtle difference between the square board and these fairly similar shapes, and it fuels hope that a simple explanation is possible. Some of the perturbed shapes are shown below.

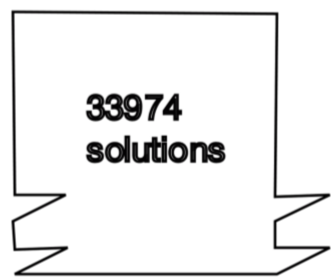


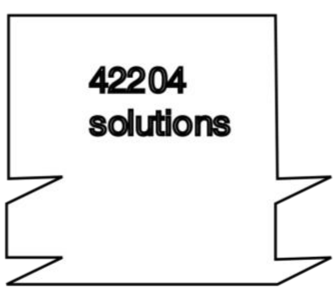
one of 189030 solutions

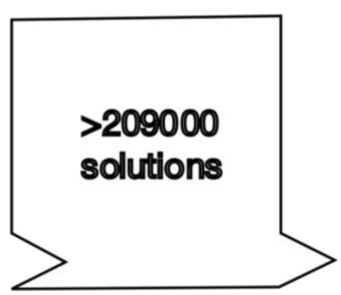


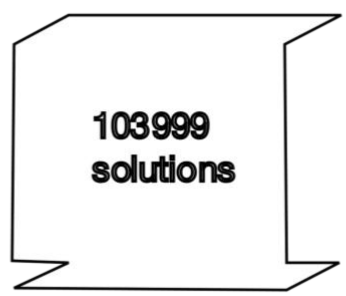




## 11. Other Rectangular Boards

There are 25 pieces, each made of 4 triangles of dimensions 2 by 1, for a total area of 100 triangles; that is, 100 half-cells, or 50 full cells. The board dimension parallel to the long leg of the component triangles must be even. This allows various rectangular boards (horizontal × vertical): 2 × 50, 4 × 25, 10 × 10, 20 × 5, 50 × 2, 100 × 1.

2 × 50 is impossible because many pieces are more than 2 units (one cell) wide.

4 × 25 is impossible because some pieces (D E S T X) are 6 units wide.

10 × 10 is impossible, by the proof herein.

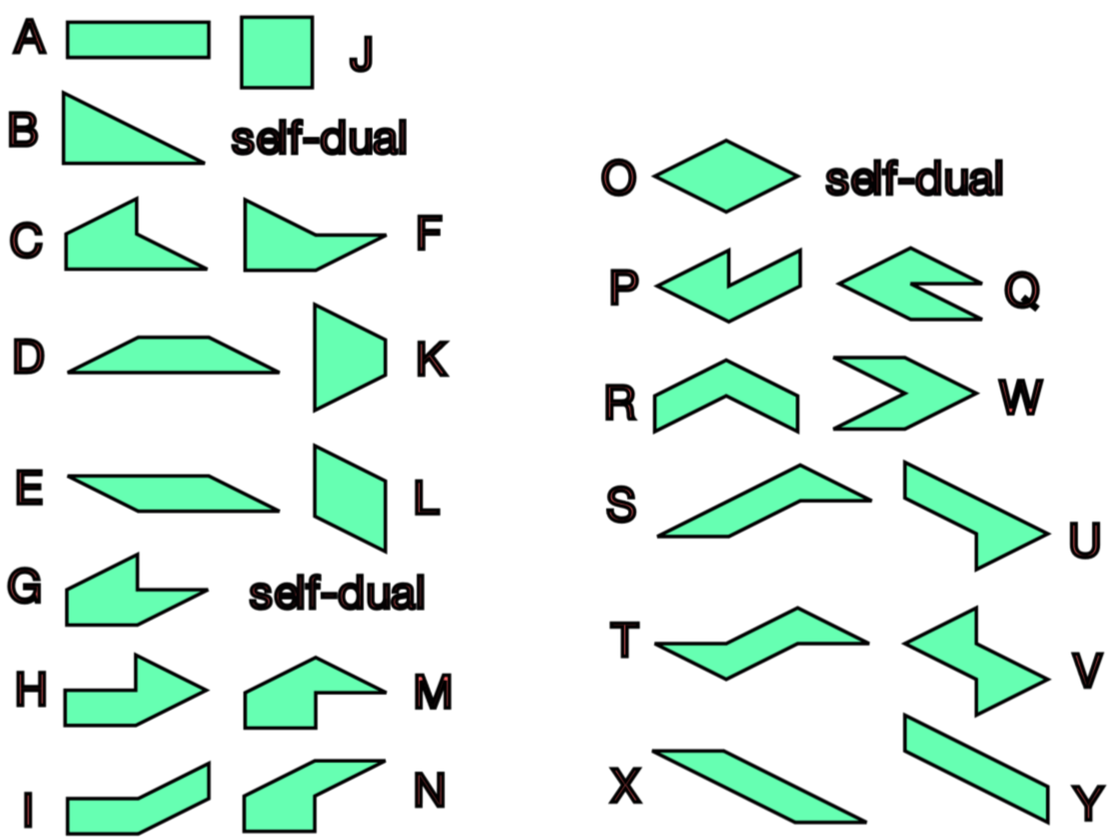
20 × 5 is impossible because it is the dual of 10 × 10 (see below).

50 × 2 and 100 × 1 are impossible because some pieces (K L U V Y) are 3 units high.

As mentioned earlier, the puzzle design distinguishes horizontal (long leg of triangles) from vertical (short leg of triangles). This distinction leads to a duality, in which left-right is exchanged with up-down. For example, instead of annexing a triangle to the (say, right) side of a full cell, annex it to the top; instead of the triangle being long on the top, make it be heavier on the right. This allows us to associate each puzzle piece with its dual. (Some pieces are self-dual.)

Further, consider the two boards that are 10×10 and 20×5 in length units. In terms of number of cells, these are (horizontal, vertical) 5×10 and 10×5 cells; they are dual boards! Because the set of all 25 tetrascrits is complete (no dual is excluded), there are solutions to either both dual boards or to neither. In this case, to neither.

The dual pairs of tetrascrit pieces are shown below.



## 12. Ideas that Did Not Work

Proofs are often presented after being honed to remove the false starts, the blundering, or the incomplete reasoning. Lest this proof give that impression, I mention some avenues that felt promising for a while but turned out to be dead ends:

- Checkerboard coloring the cells.
- Drawing both diagonals of the cells, and checkerboard coloring the quarter-cells.
- Calculating the perimeters of pieces.
- Border pieces that "overhang" along the border, and which pieces "under-hang" so they can be positioned to fill in the overhang. In this vein, which pieces can follow which other pieces along the border, the "graph of succession", and limits it might impose.
- Count of half-cells.
- Which pieces can occur around the entire border: vertical sides, horizontal sides, corners.
- Vertical and horizontal "dotted lines" within pieces, where the rectangular grid lines lie.
- Horizontal bands of alternating color.
- Diagonal bands of alternating color.
- Coloring the board with 4 colors per group of 2x2 cells.

## 13. Other Polyscrit Piece Sets

If we use a different set of polyscrit pieces, can they make a rectangle? Puzzle variations common in the tiling community include considering a different order (size of pieces), or the collection of all pieces from order 1 through some order "n", or different shapes of board, etc. Some amazing results, using other polyforms, include boards that are several copies of one shape; enlarged versions of the pieces themselves; nested rings; and so on. Designers also turn to one-sided pieces to get total areas that equate to interesting shapes.

Here, we keep to a simple regimen: 2-sided polyscrits on a rectangular board. For a given order, we use all the pieces or none. We dislike pieces with holes, so we do not use pieces of order above 6. We assume the piece counts in the next section are accurate. The table below shows there are not many interesting cases. The only nice candidate is orders 1 through 5; can the 94 pieces fit in an 18 by 23 rectangle?

| Order | Pieces | Total Area (half-cells) | Comments |
|---|---|---|---|
| 1 | 1 | 1 | impossible (odd) |
| 2 | 5 | 10 = 2×5 | impossible |
| 3 | 6 | 18 = 2×3×3 | impossible |
| 4 | 25 | 100 = 2×2×5×5 | impossible (per this paper) |
| 5 | 57 | 285 = 3×5×19 | impossible (odd) |
| 6 | 214 | 1284 = 2×2×3×107 | awkward (107 is big) |
| 1 – 2 | 6 | 11 | impossible (odd) |

| | | | |
|---|---|---|---|
| 1 – 3 | 12 | 29 | impossible (odd) |
| 1 – 4 | 37 | 129 = 3×43 | impossible (odd) |
| 1 – 5 | 94 | 414 = 2×3×3×23 | maybe 18×23 |
| 1 – 6 | 308 | 1698 = 2×3×283 | unlikely (very narrow) |

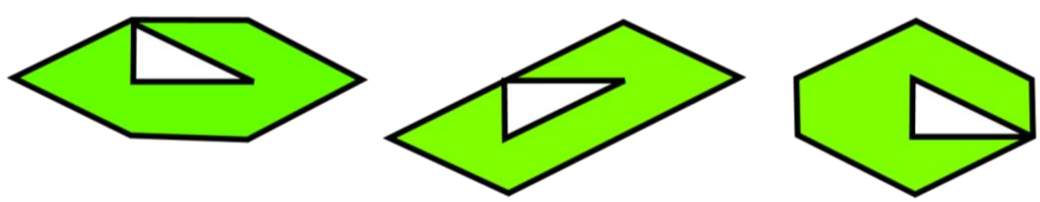
## 14. Polyscrit Counts

An important aspect of any sort of polyform is how the number of distinct pieces changes with the order, the number of basic units joined. Although I am confident that my manual construction of orders 1 through 4 polyscrits is correct, both the task and the risk of error grow quickly as the order increases. Consequently, I wrote a program to count them. Its results are below.

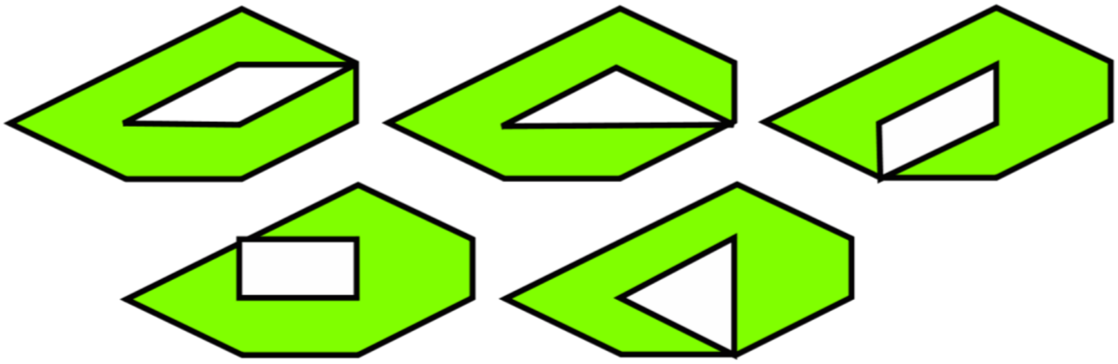
| Order | Count of Polyscrits | Enclosing Holes | Hole Size 1 | Hole Size 2 | Hole Size 4 | Count Ratio |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | — | — | — | — |
| 2 | 5 | 0 | — | — | — | 5 |
| 3 | 6 | 0 | — | — | — | 1.2 |
| 4 | 25 | 0 | — | — | — | 4.17 |
| 5 | 57 | 0 | — | — | — | 2.28 |
| 6 | 214 | 0 | — | — | — | 3.75 |
| 7 | 665 | 3 | 3 | 0 | 0 | 3.11 |
| 8 | 2425 | 22 | 22 | 0 | 0 | 3.65 |
| 9 | 8456 | 148 | 142 | 5 | 0 | 3.49 |
| 10 | 31028 | 857 | 804 | 51 | 2 | 3.67 |

These counts are for two-sided pieces; that is, the reflection of a piece is considered the same piece. Hole size is the hole area in half-cells. "Count of Polyscrits" includes those with holes.

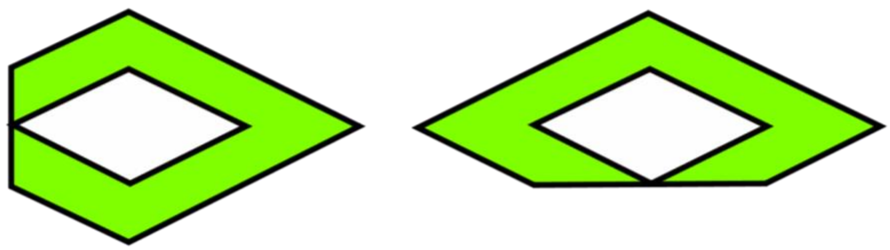
Another important metric is how many pieces enclose one or more holes. This is especially important in tiling problems, because the holes cannot, in general, be filled by other pieces. For polyscrits, the lowest order with holes is order 7. The 3 order-7 holey pieces are shown below.



The 5 order-9 pieces with a hole size of 2 half-cells are below. Curiously, they are all the same external shape, with holes the shape of the 5 order-2 polyscrits.



The two order-10 pieces with a hole area of 4 half-cells are below. They are a dual pair. Strangely, size 3 hole is skipped. It will appear in order-11 pieces, when the eleventh piece fills in part of the hole in a piece below.



Finally, the ratio of the count for one order to that of the previous order, is an interesting metric of how fast the count increases. From the "Count Ratio" column above, the ratio seems maybe converging on roughly 11/3.

The values in the above table have not been independently verified. In my experience, programs for such counts often have bugs. Caveat emptor! I would appreciate any corroboration or correction to the numbers.

## 15. One-sided and Translation-only Variants

A census of polyscrits would be improved by brief mention of one-sided and translation-only variants. All discussion above applies only to two-sided pieces. One-sided pieces are considered distinct if turning them over (reflection) creates a new shape. Most of the 25 two-sided, order 4 polyscrits, the tetrascrits, have a different shape when flipped over. Seven have the same shape when flipped: the dual pairs A and J, D and K, R and W; and O (self-dual). The other 18 are distinct when flipped, for a count of 2×18=36, plus those 7, yields 43 one-sided order-4 pieces, called "1s-polyscrits".

If we also disallow turning a piece 180 degrees, we have pieces that can be moved around the board only by translation. This constraint is not common in tiling and puzzling, but it is the logical end of restricting the orientation of pieces. The table below gives statistics for both one-sided (1s-polyscrits) and translation-only (xo-polyscrits) variants.

| | Polyscrits | | One-sided (1s-polyscrits) | | Translation-only (xo-polyscrits) | |
|---|---|---|---|---|---|---|
| **Order** | **Count** | **Enc. Holes** | **Count** | **Enc. Holes** | **Count** | **Enc. Holes** |
| 1 | 1 | 0 | 2 | 0 | 4 | 0 |
| 2 | 5 | 0 | 7 | 0 | 9 | 0 |
| 3 | 6 | 0 | 12 | 0 | 24 | 0 |
| 4 | 25 | 0 | 43 | 0 | 71 | 0 |
| 5 | 57 | 0 | 114 | 0 | 228 | 0 |
| 6 | 214 | 0 | 408 | 0 | 768 | 0 |
| 7 | 665 | 3 | 1330 | 6 | 2660 | 12 |
| 8 | 2425 | 22 | 4787 | 44 | 9407 | 88 |
| 9 | 8456 | 148 | 16912 | 296 | 33824 | 592 |
| 10 | 31028 | 857 | 61847 | 1708 | 123093 | 3416 |

Of course, the counts of one-sided pieces are at most 2 times the "plain" polyscrits, and the counts of translation-only pieces are at most 4 times. They are slightly less than those upper bounds because of pieces with symmetry.

Note that for odd orders, the counts of one-sided pieces are exactly 2 times the "plain" counts, and the counts of translation-only pieces are exactly 4 times. I believe that this is because odd order polyscrits cannot have reflection or rotation symmetry, a result of the basic unit triangle being asymmetric.

The counts of pieces with holes have ratios 1:2:4 in the above table (except for order 10 "plain" polyscrits: 857 instead of 1708/2=854). I assume this is due to symmetric pieces, and I expect that for larger even orders, all the holey counts will diverge from 1:2:4.

Can one-sided or translation-only polyscrits pack rectangles? Cases analogous to the "plain" pieces are summarized below, with my guess about the chances of solutions. They are not very promising.

| Order | Pieces | Total Area (half-cells) | Comments |
|---|---|---|---|
| 1 | 2 | 2 | impossible |
| 2 | 7 | 14 = 2×7 | impossible |
| 3 | 12 | 36 = 2×2×3×3 | unlikely |
| 4 | 43 | 172 = 2×2×43 | very unlikely |
| 5 | 114 | 570 = 2×3×5×19 | maybe, but big |
| 6 | 408 | 2448 = 2×2×2×2×3×3×17 | maybe, but very big |
| 1 – 2 | 9 | 16 = 2×2×2×2 | very unlikely |
| 1 – 3 | 21 | 52 = 2×2×13 | very unlikely |
| 1 – 4 | 64 | 224 = 2×2×2×2×2×7 | maybe, but big |
| 1 – 5 | 178 | 794 = 2×397 | impossible |
| 1 – 6 | 586 | 3242 = 2×1621 | impossible |

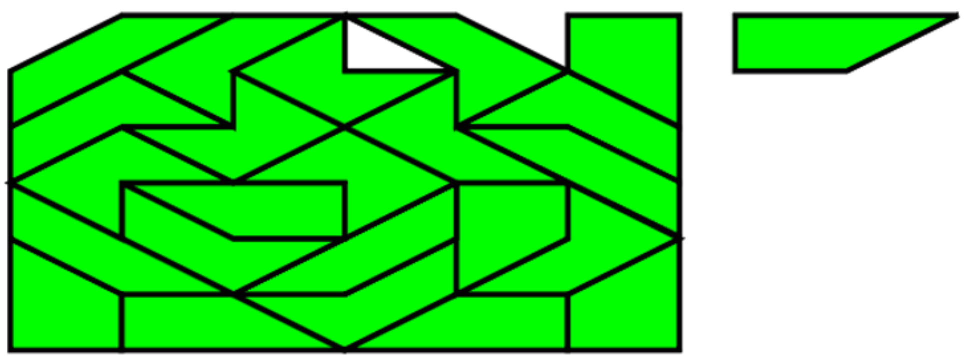
**Potential rectangles with one-sided polyscrits.**

| Order | Pieces | Total Area (half-cells) | Comments |
|---|---|---|---|
| 1 | 4 | 4 = 2×2 | yes, trivial |
| 2 | 9 | 18 = 2×3×3 | unlikely |
| 3 | 24 | 72 = 2×2×2×3×3 | no (see text) |
| 4 | 71 | 284 = 2×2×71 | very unlikely |
| 5 | 228 | 1140 = 2×2×3×5×19 | maybe, but big |
| 6 | 768 | 4608 = 2^9 × 3^2 | maybe, but very big |
| 1 – 2 | 13 | 22 = 2×11 | impossible |
| 1 – 3 | 37 | 94 = 2×47 | impossible |
| 1 – 4 | 108 | 378 = 2×3×3×3×7 | maybe |
| 1 – 5 | 336 | 1518 = 2×3×11×23 | maybe, but big |
| 1 – 6 | 1104 | 6126 = 2×3×1021 | very unlikely |

**Potential rectangles with translation-only polyscrits.**

## 16. Closing the Circle

Perhaps the reader would like to try their hand at a proof of impossibility. The set of 24 translation-only triscrits, with a total area of 72, has several potential rectangles. An exhaustive computer search finds none have any solutions. The closest to having a solution is 12×6, which can be filled with 23 pieces in several dozen ways, leaving various different pieces out, such as the example below. Can you find an explanation why the 12×6 rectangle is impossible?



**Not in OEIS**
As of 30 March 2026, the counts of "plain", one-sided, and translation-only polyscrits are not in the Online Encyclopedia of Integer Sequences. The closest seems to be A353978, "Number of fixed polytans (polyaboloes) with n cells." It is 4, 9, 24, 71, 224, 740, …, and agrees with my count of translation-only polyscrits in the first four terms. My count has 228 instead of 224. Perhaps I have a bug in my counting program, or maybe they are different sequences.

Revision history:
Version 1, 7 March 2026; a false start, a flawed proof.
Version 2, 12 March 2026; revised approach, hopefully correct proof.
Version 3, 15 March 2026; note terms “dom” and “polydom” are used in the literature.
Version 4, 21 March 2026; add counts for orders 4 through 10.
Version 5, 24 March 2026; add other piece sets; 1-sided; translation-only.
Version 6, 2 April 2026; improve one-sided pieces discussion.

——— The end. ———