A Pleasing Random Block Tile Pattern
or
Teleology of Bathroom Floor Tile

Version 3, December 2025
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A noticeable number of tiled floors in the New England area are covered with the pattern shown in Figure 1a. The pattern is used with translation only (Figure 1b), not rotation or reflection, to tessellate floors in both commercial buildings and private homes, especially bathroom floors. On some floors, the mirror image of Figure 1a is used instead. Clearly, this pattern was sold and installed in great quantity. The pattern looks fairly random; no regularity jumps out at the eye, even in the repetition on a floor. What is it about this pattern that gives it its random appearance? That is the subject of this paper.

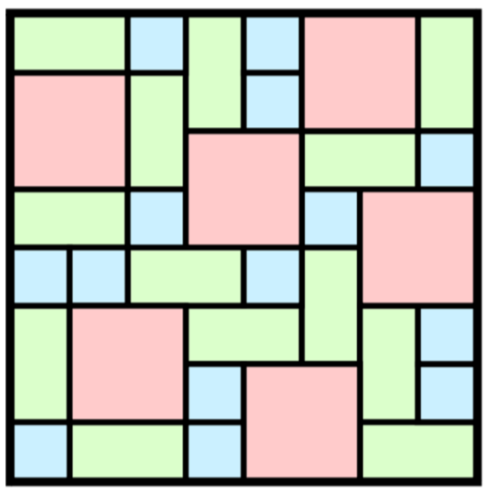


Figure1a. A "random block" tile pattern. Color is incidental; typical floors are monochrome.

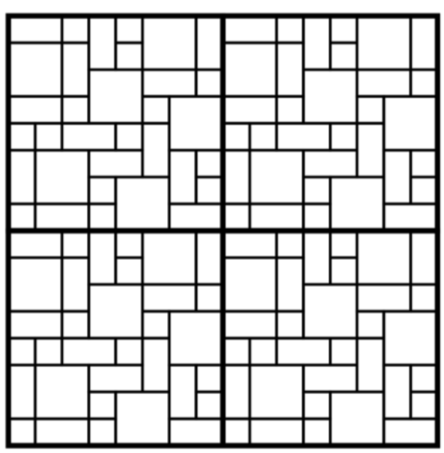


Figure 1b. The popular floor tile pattern.

Probably the pattern was designed to look pleasantly random. If so, it is an example of a tension between order and disorder that often arises in art. Where is the most pleasing point in that space between order and chaos? The design in Figure 1 is a context in which we can investigate this question.

to be supplied: What company(s) produced this pattern? During what years? Was it marketed outside of New England? Can I find out who designed it, and perhaps even what their thoughts were in choosing this pattern?

Further, we can ask whether Figure 1 is just one of many patterns that would serve equally well, or does it alone satisfy some aesthetic criteria of randomness? I hypothesized rules for a tile pattern to meet so that it would appear random, and used a computer to find all patterns satisfying those rules. After some trial and error, I arrived at a set of rules that only Figure 1 satisfies. Some of these rules you will probably agree with; others you may think are contrived, arbitrary. To the extent that you agree with them, Figure 1 is, in a practical sense, a uniquely random tiling.

Rule 1:
We are concerned only with tile shape, not color, texture, etc.

Rationalization:
Known floors tiled with the Figure 1 pattern do not have distinctive colors, textures, etc.

Rule 2:
Each tile shape is a rectangle with integer dimensions.

Rationalization:
Rectangular ceramic tiles are easy to make, will pack to cover a plane, and are relatively resistant to cracking.

Rule 3:
Only a few different shapes will be used. Namely, 1x1, 1x2 and 2x2.

Rationalization:
Having only one or two shapes does not permit enough variety of pattern. Three shapes give much greater variety, and these are the smallest (and therefore simplest) three. Simplicity is evident since the 1x1 and 2x2 have only one orientation, and the 1x2 has only two (horizontal, vertical).

Rule 4:
The pattern that is repeated to cover the floor will be 8x8 units.

Rationalization:

When our eye searches the floor for a pattern, a tessellation of less than about 8x8 stands out. More than about 8x8 is wasted complexity in the manufacturing process. Smaller and larger floor tile patterns are used, but 8x8 seems to be a good compromise. Also, having a square pattern helps avoid making the horizontal direction look any more or less random than the vertical direction.

The next few rules govern the placement of 2x2 tiles within the 8x8 block. After that placement is resolved, we will turn to filling the remaining space with 1x1 and 1x2 tiles.

Rule 5:
2x2 tiles can touch at their corners, but must not touch along their sides, within the border of the 8x8 block.

Rationalization:
If 2x2 tiles can touch along their sides, they appear as a clump, reducing the appearance of randomness. A 2x2 can touch the side of another 2x2 across the border in another 8x8 block, because the unbroken border line visually breaks up that clump.

Rule 6:
No two 2x2 tiles can be centered in the same row or column.

Rationalization:
Vertically or horizontally aligned 2x2 tiles within the 8x8 block create a sense of grouping, especially since they are also aligned with their corresponding 2x2s in neighboring 8x8 blocks.

Rule 7:
It is desirable to have as many 2x2 tiles as possible.

Rationalization:
If there are only a few 2x2 tiles, they will be conspicuous. Their anomalous appearance will detract from the sense of randomness.

The computer was used to find all ways to place 2x2 tiles that satisfy the above rules. The maximum number of 2x2 tiles is seven, and there is a unique placement, shown in Figure 2. (Since there is no special "north" side of a floor, rotations of the pattern are not considered distinct.)

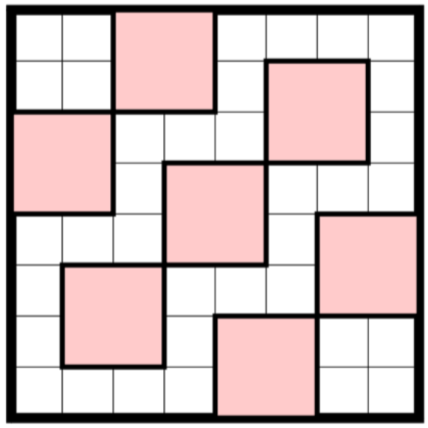


Figure 2. Placement of seven 2x2 tiles.

The pattern in Figure 2 looks highly structured, not random. We need a rule to reject it. Then we will turn to patterns with only six 2x2 tiles, and seek further rules.

Rule 8:
The placement of the 2x2 tiles cannot have symmetry along either diagonal of the 8x8.

Rationalization:
The 2x2 tiles are so visually distinct that, no matter how the rest of the pattern is filled in, such symmetry will stand out. This also eliminates some patterns with six 2x2 tiles, such as Figure 3.

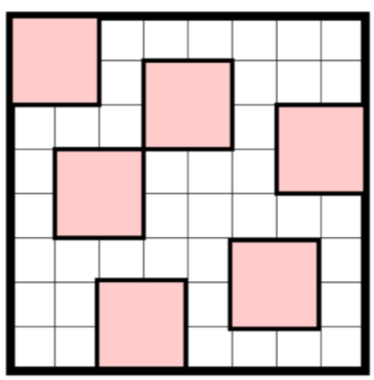


Figure 3. Six 2x2 tiles with an axis of symmetry.

Rule 9:
Within the 8x8 pattern, the smallest possible number of 2x2 tiles may touch corner-to-corner.

Rationalization:
The 2x2 tiles stand out so much that if more than one pair touches corner-to-corner, they become a focus of attention. One pair may so touch and still look accidental.

It happens that no pattern (with six 2x2 tiles) has no corner-to-corner touching at all. The minimum touching is one pair of 2x2 tiles. This eliminates patterns with three 2x2 tiles in a diagonal line as in Figure 4, and double pairs of tiles as in Figure 5.

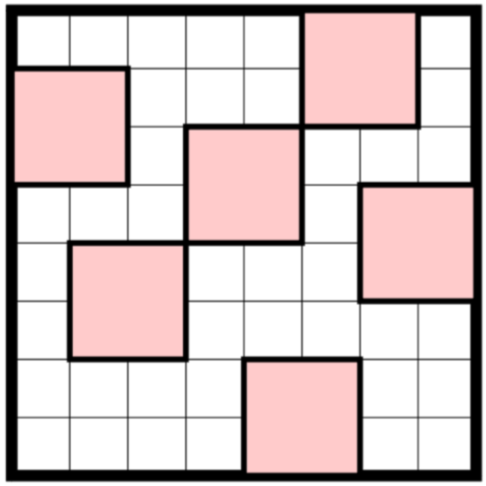


Figure 4. Six 2x2 tiles with three in a diagonal line.

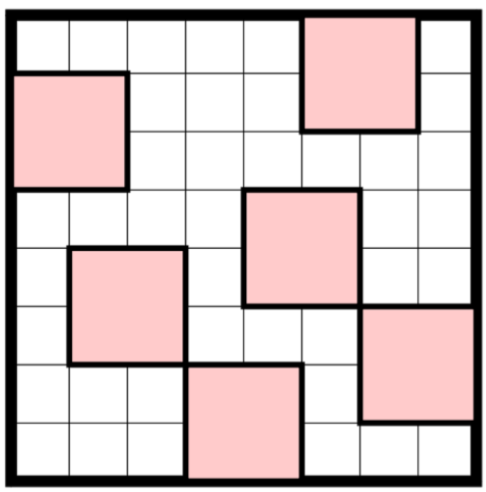


Figure 5. Six 2x2 tiles with two pairs.

The above rules suffice to eliminate all patterns with six 2x2 tiles except the one in Figure 1. So far, we have succeeded in finding rules that permit only that pattern. What remains is to find rules that restrict the placement of 1x1 and 1x2 tiles in the remaining space.

Rule 10:
Pairs of 1x2 tiles may not lie side-by-side or end-to-end, either within the 8x8 block or across its border, either vertically or horizontally.

Rationalization:

1x2 tiles catch the eye if they are stacked side-by-side or aligned end-to-end. They appear related to each other, not just coincidentally falling there. However, 1x2 tiles may touch along half their sides without looking too structured. Figure 6 shows several violations of this rule.

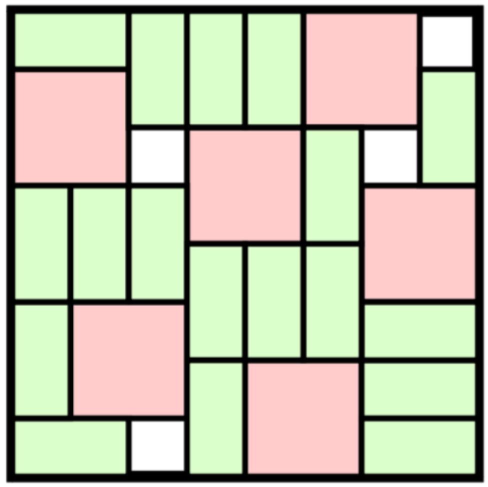


Figure 6. 1x2 tiles stacked or in line.

Rule 11:
Triplets of 1x1 tiles may not be rook-wise connected, either in a line or bent, either within the 8x8 block or across its border, in any orientation.

Rationalization:
1x1 tiles joined in a group vertically and/or horizontally (as a rook moves in chess) catch the eye and do not look random. The larger the group, the more structured it looks. Insisting that each 1x1 be alone is too strict, so pairs of 1x1 tiles are permitted, but triplets are prohibited. Figure 7 shows, within the 8x8 border, one instance of each illegal group, plus one legal pair of 1x1 tiles. When this pattern tessellates a floor, the 1x1 groups in the lower left and lower right corners will connect across the border, forming an even larger (and thus even worse) group.

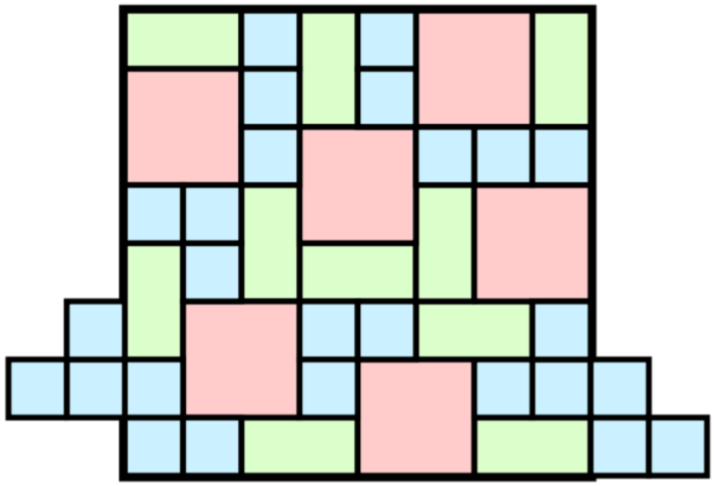


Figure 7. Groups of three 1x1 tiles.

At this point, I wrote a computer program to count the 8x8 patterns that satisfy all the above rules. There are 69,678. I then used the program to explore the effect of various rules in filtering out all patterns except that of Figure 1. The result is the rules below.

Rule 12:
The 8x8 pattern must have no horizontal or vertical break.

Rationalization:
If one of the seven horizontal lines running across the 8x8 has no tile crossing it, then the 8x8 visually and physically falls into two rectangles. Also, when the 8x8 tessellates the floor, there would be visual ambiguity about the identity of the repeated pattern; it would look like two different patterns forming alternating stripes. The same argument applies to a vertical break.

In Figure 1, the 2x2 tiles fall across six of the seven horizontal lines and six of the seven vertical lines, so there is only one possible place for a break in each direction. For example, Figure 8 has both a horizontal break and a vertical break.

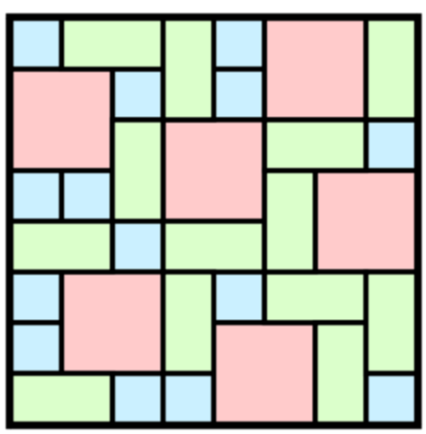


Figure 8. Horizontal and vertical breaks.

Rule 13:
The almost-breaks must be spanned by the minimum structure.

Rationalization:
Although a full break is very noticeable visually, the long lines in the floor due to the border of the 8x8 block are also noticeable. Those border lines will stand out less if there are other long lines to distract our eye from them. Figure 1 contains only two possible positions for lines that are maximally long but not complete breaks. Each of these should be spanned by a single 1x2. Figure 9 shows a pattern with two 1x2 tiles (not just one) across both the horizontal and the vertical almost-breaks.

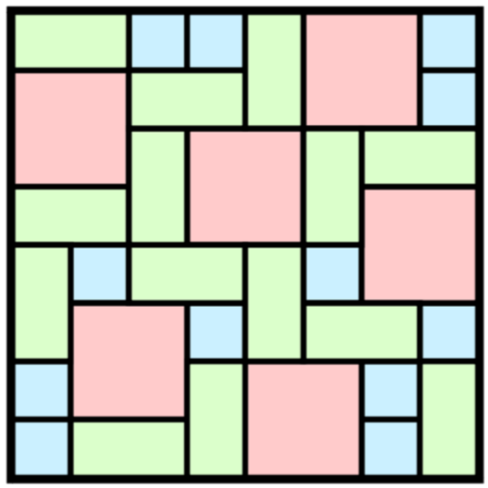


Figure 9. Double spanning of almost-breaks.

Rule 14:
The total areas used by tiles of identical shape must be balanced.

Rationalization:
A pattern dominated by 1x1 tiles, by horizontal 1x2 tiles, or by vertical 1x2 tiles will not look random. The predominant shape, here including orientation, will establish a visual background texture, and the minority shapes will stand out as rare. Total area is a better measure of balance than tile count, because without color or texture to emphasize the small shapes, they are dominated by larger shapes of equal number.

40 square units of area are to be divided among three shapes, as equally as possible. Exact equality is impossible. The closest divisions are:

	area (not number) of tiles		
shape of tile	**case 1**	**case 2**	**case 3**
horizontal 1x2	12	14	14
vertical 1x2	14	12	14
1x1	14	14	12

Because horizontal 1x2 tiles and vertical 1x2 tiles are both 1x2 in shape, there is some perceptual weight that each will gain by the presence of the other. Therefore, case 3 is less balanced than cases 1 and 2, because in case 3 the area of 1x1 tiles is exceeded not only by the area of each of the other two, but further by their synergistic effect on each other. Therefore, we interpret the balance requirement to mean case 1 or 2. We do not discriminate between cases 1 and 2, in accordance with wanting no preferred orientation of the 8x8 pattern.

As examples of how out of balance an otherwise fairly random-looking pattern may be, Figure 10 has one horizontal 1x2 tile and eleven vertical 1x2 tiles, while Figure 11 has the reverse.

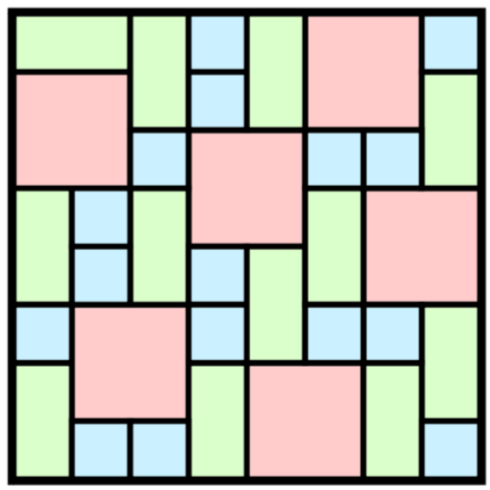


Figure 10. One horizontal, eleven vertical 1x2 tiles.

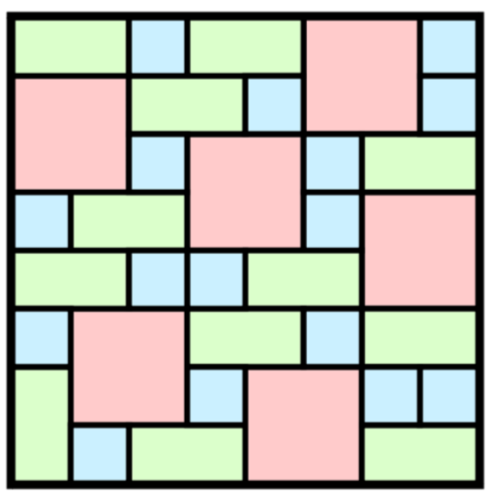


Figure 11. Eleven horizontal, one vertical 1x2 tiles.

Rule 15:
Within the 8x8 pattern, certain structures must not occur in 2x3 areas.

Rationalization:
If 1x2 and 1x1 tiles form a symmetrical group within a 2x3 area, the eye will pick up that area as an island of structure within the overall random pattern. We allow formation of such structures across the 8x8 boundary because the boundary itself helps disrupt the structured appearance. We identify two prohibited structures, shown in Figure 12. Three of one and one of the other occur in the 8x8 pattern of Figure 13.

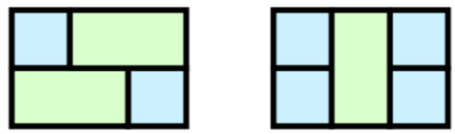


Figure 12. Two prohibited 2x3 structures.

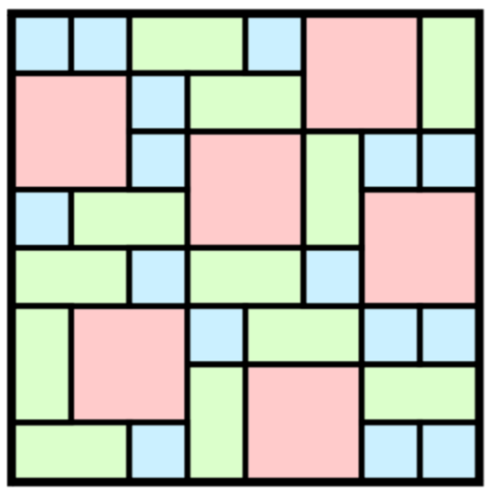


Figure 13. 8x8 pattern with bad 2x3 structures.

Rule 16:
Within the 8x8 pattern, pairs of 1x1 tiles must occur in both the horizontal and the vertical orientation.

Rationalization:
If all pairs have the same orientation, the pattern does not look random. Further, a pattern with no pairs at all looks contrived, as if the 1x1 tiles are purposely separated from each other. Figure 14 shows eight horizontal and no vertical pairs, and Figure 15 shows the reverse. The pattern in Figure 16 has no pairs.

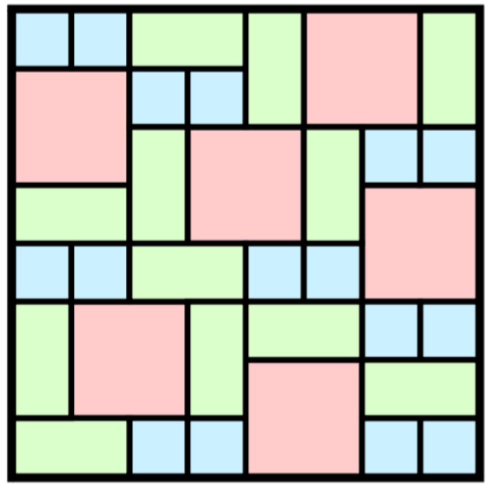


Figure 14. Eight horizontal 1x1 pairs.

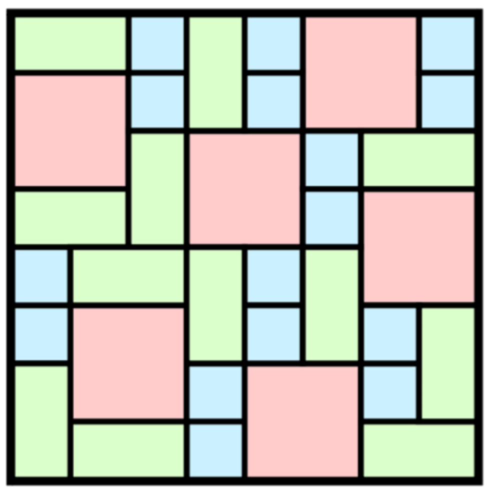


Figure 15. Eight vertical 1x1 pairs.

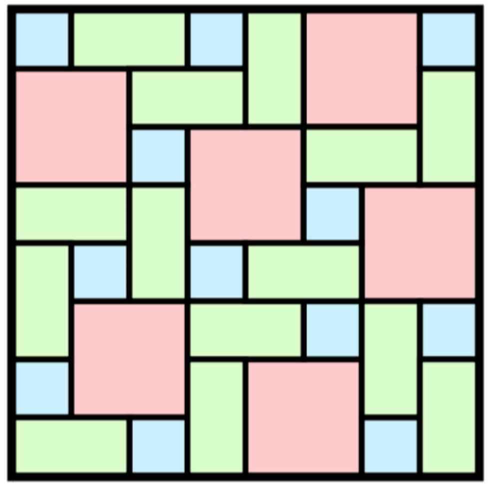


Figure 16. No 1x1 pairs.

Rule 17:
At least four pairs of 1x1 tiles must touch the border of the 8x8 pattern.

Rationalization:
If too many pairs are concentrated inside the 8x8 pattern, they draw attention to the center of the pattern and thus increase the perception of separate patterns forming a tessellation, instead of randomness on a large scale. Figure 17 shows a pattern with five internal pairs and none touching the border. Requiring that (at least) four touch the border is somewhat arbitrary, but is not close to the maximum attainable; Figure 18 shows a pattern with nine pairs, seven of which touch the border.

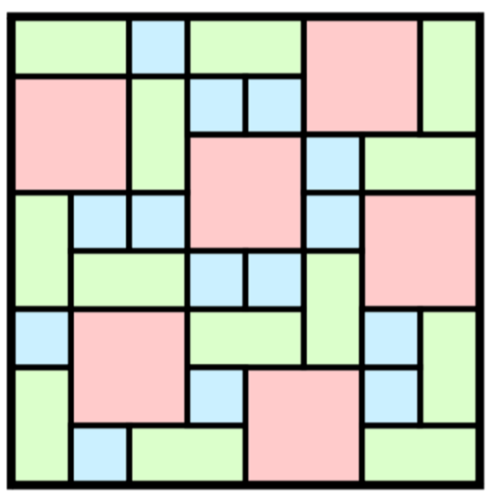


Figure 17. All 1x1 pairs are internal.

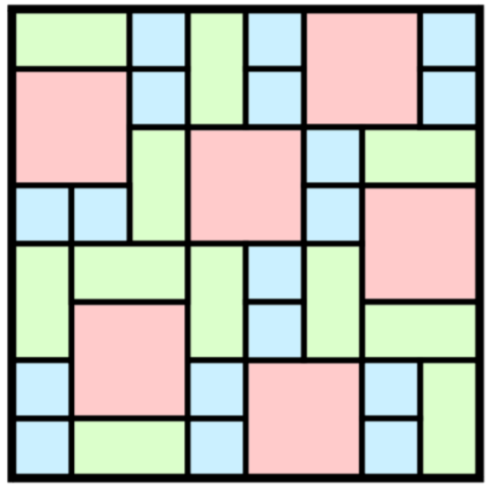


Figure 18. Seven 1x1 pairs touch the border.

Rule 18:
Neither diagonal of the 8x8 pattern may have more than one 1x1 tile on it.

Rationalization:
If many 1x1 tiles are aligned diagonally, an appearance of diagonal lines will arise. Since the diagonals of the 8x8 pattern are intrinsically visually important, 1x1 tiles especially stand out there. Figure 19 shows a pattern with five 1x1 tiles on one diagonal and three on the other.

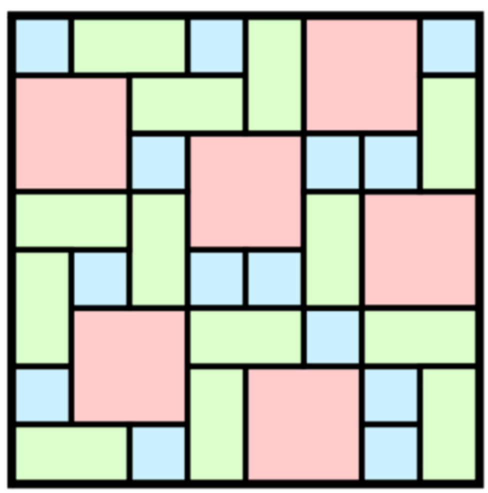


Figure 19. Several 1x1 tiles on the diagonals.

At this point, Figure 1 is the only pattern that satisfies all the rules.

I find the last few rules rather contrived, but not outrageously so. Some of them seem a bit unlikely as a priori constraints that a person would place on patterns, but believable as constraints arising during a search for a pattern that looks random. The discussion below may help you decide for yourself how contrived you think the above rules are.

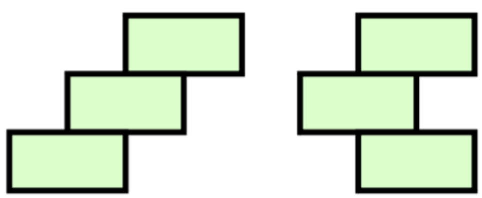
Rules 12 through 18 seem the most contrived, so I looked closely at the effect each has on filtering out patterns. The results are summarized below. For each of the seven rules, the table shows how many patterns (including Figure 1) remain (of the 69,678) if that rule alone is removed from the eighteen rules; also, how many patterns remain if only the given rule, plus the basic rules 1 through 11, are used.

	number of 8x8 patterns that result from using…	
rule	**all 18 except this rule**	**rules 1-11 and this rule**
12 no breaks	4	31811
13 two almost-breaks	13	45659
14 areas balanced	7	16116
15 no 2x3 structures	32	26198
16 >0 h, v 1x1 pairs	2	53420
17 >3 edge 1x1 pairs	15	12305
18 <2 1x1s per diagonal	346	3607

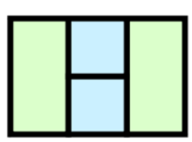
For example, using the 18 rules, rule 16 is needed only to eliminate a single pattern, that obtained from Figure 1 by turning the horizontal pair of 1x1 tiles and the 1x2 tile next to it clockwise a quarter turn.

I considered, but rejected, the following rules. They were not necessary beyond the rules above to filter out patterns.

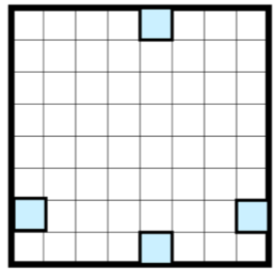
- **Rule A**: Three 1x2 tiles may not occur side-to-side along a diagonal, either within the border of the 8x8 or in adjoining 8x8 areas. Two examples are:



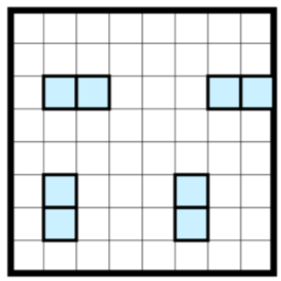
- **Rule B**: The following 2x3 sub-pattern may not occur within the 8x8 pattern:



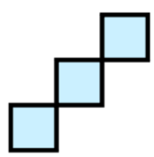
- **Rule C**: A pair of 1x1 tiles may not occur split across the border of the 8x8 pattern. This example has two such pairs:



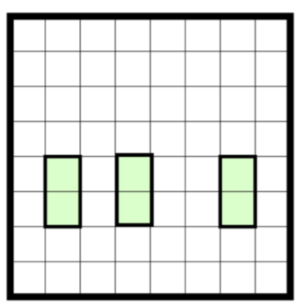
- **Rule D**: Two pairs of 1x1 tiles may not occur in the same row(s) or column(s) within the 8x8 pattern, with both pairs either horizontal or vertical, and with the relative position either sideways or endwise. Such occurrence tends to look like a stacking of the pairs. Examples:



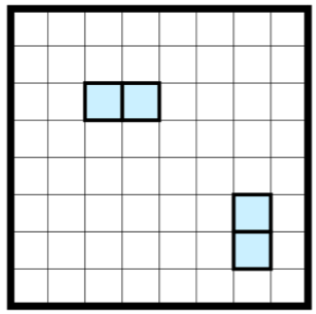
- **Rule E**: Three 1x1 tiles may not occur adjacent along a diagonal line, either within the 8x8 pattern or across its border. For example:



- **Rule F**: Three horizontal 1x2 tiles may not occur in the same two columns, and three vertical 1x2 tiles may not occur in the same two rows. For example:



• **Rule G**: Every pair of 1x1 tiles must touch the border. An example of two pairs, each completely within the 8x8 block, is:



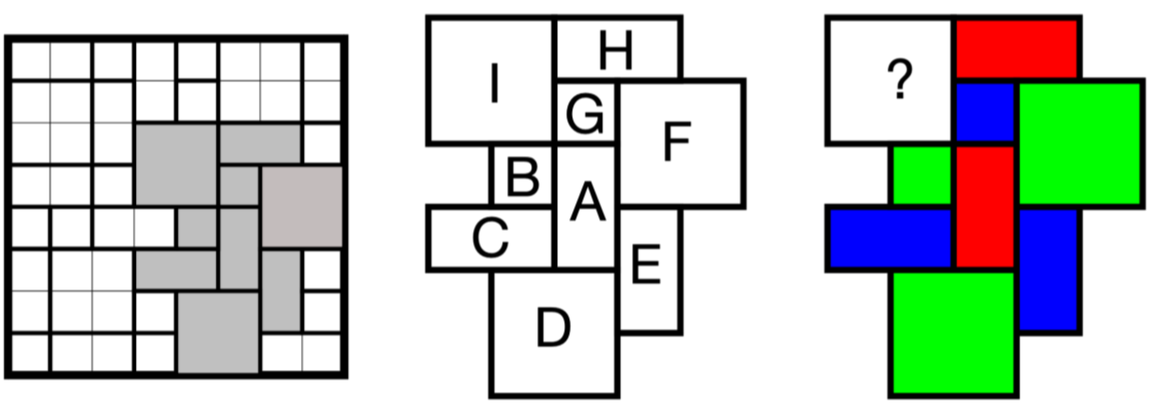
• **Rule H**: The total area of 1x1 tiles must be divided among single tiles and pairs as evenly as possible. Figure 1 has:

6 single 1x1s, for area 6
1 horizontal 1x1 pair, for area 2
3 vertical 1x1 pairs, for area 6

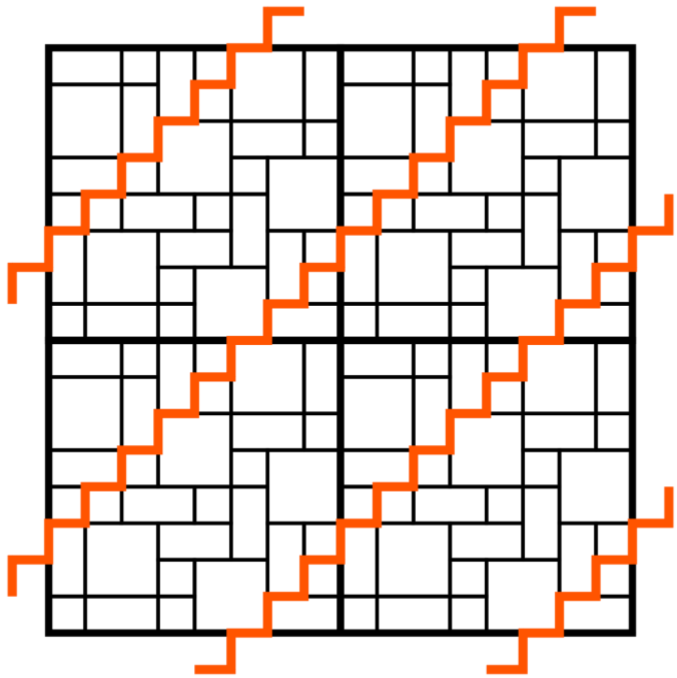
The total area of 1x1 pairs is 8, versus 6 for single 1x1s. If one pair were instead two single 1x1s, the result would be 6 versus 8, the same imbalance. Any other split would be more imbalanced.

• **Rule I**: The 8x8 pattern must not be 3-colorable; that is, coloring the tiles so that no tiles of the same color touch along their edges, must require four colors.

The pattern in Figure 1a requires four colors, as follows. Let tile A be red. A is surrounded by B through G, so they must alternate two new colors; say B = D = F = green, C = E = G = blue. Tile H touches both F and G, so it must be red. Tile I touches tiles B = green, G = blue, and H = red, so it requires a fourth color.



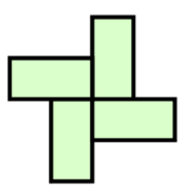
• **Rule J**: A stairstep (zigzag) break exists in the full tessellation. The break in Figure 1 is shown below. It crosses each 8x8 in two places because it is not on a main diagonal.



• **Rule K**: All 12 pentominoes can be formed by selecting specific tiles in the pattern. (Any reflection or orientation is allowed, and crossing the 8x8 border is allowed). (All smaller polyominoes are also present, but that is unsurprising.) The difficulty of spotting pentominoes seems to testify to the randomness of the pattern. A manual count finds these numbers of distinct pentominoes in Figure 1a, contained within or crossing the 8x8 border:

pentomino	F	I	L	N	P	T	U	V	W	X	Y	Z
within 8x8	2	1	2	4	15	1	1	5	3	0	3	1
crossing	3	1	5	3	7	1	2	5	2	1	5	1

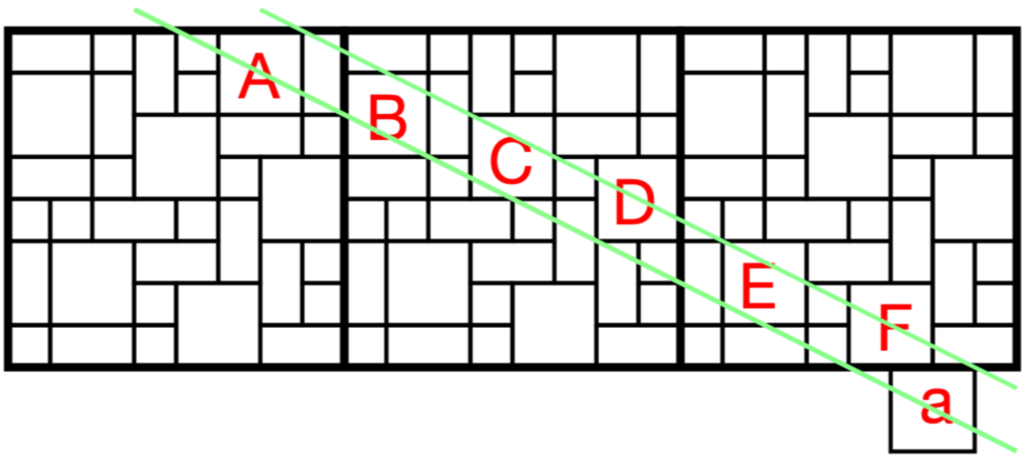
• **Rule L**: There is no pinwheel at the four corners intersection. Here, a "pinwheel" is four 1x2 tiles placed like blades of a fan. Figure 1 has three 1x2s in pinwheel places, but its bottom left corner is a 1x1. If it were a vertical 1x2, it would make this pinwheel, and that would look very non-random:



• **Rule M**: In the full tessellation, a slanted line can be drawn that passes through each of the six 2x2s (some in different 8x8s), and continues to do so indefinitely. (Informally, the 2x2s A through F in the diagram below seem to be in a line.)

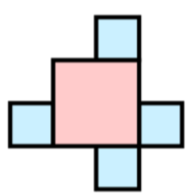
Consider the diagram below, showing some of the 8x8s in three copies of Figure 1a. The 2x2s B and C are offset by 1 vertically and 3 horizontally, as are C and D (and also E and F). Thus B, C and D are lined up along a sloping line. If this line is extended to the left, it comes close to A in the neighboring 8x8. Extending it to the right, it comes close to E and F in the other 8x8. All six 2x2s are touched, before a second copy of any of them is touched.

For this process to work indefinitely, the line must have slope -1/2, falling 1 unit vertically for each 2 units horizontally. There is a band of lines that work, bounded above by the top right corners of 2x2s A and a, and bounded below by the bottom left corners of 2x2s D and F.



(It may be that this band of lines is not unique, and other lines intersect all six 2x2s in a similar way.)

• **Rule N** (suggested by Hilarie Orman): There is a 2x2, each side of which contacts a 1x1 within the 8x8. Figure 1 contains this sub-pattern:



Revision history:
Version 1, 30 May 1990; end of original investigation.
Version 2, 20 November 2022; minor fixes, add examples of rejected rules, add more rejected rules.
Version 3, 2 December 2025; convert to Word, with Inkscape for diagrams.

——— The end. ———